## Tactical Level in SARNET Overview and Transitions between Equilibria

#### SNE, The University of Amsterdam, The Netherlands



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- Study and use the alliance
- Improve and reorganise cooperation
- Implement the strategies

These topics inspire the following

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  - Studying and using the alliance
    - How to motivate cooperation?
    - Whom to ask for help?
    - What are the possible problems?
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## The Goals

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  - e How to move to a better equilibrium?



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- Improving and reorganising the cooperation
- Implementing the strategy
  - Filtering flows
  - Oeleting links



Measuring the efficiency of defence

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Algorithm based on trust and the importance of the defence

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Filtering undesirable flows to allow the desirable flows utilise the network fully  $% \left( {{{\left[ {{{L_{\rm{B}}} \right]}} \right]}_{\rm{B}}}} \right)$ 

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#### Implementing the strategy

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Deleting links -Removing undesirable flows by deleting edges



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## Solutions, Transitions and Efficiency - Introduction

We study how lack of coordination (domains, etc.) influences efficiency

#### Definition

A game  $(N, S = S_1 \times ... \times S_N, u_1, ..., u_n)$  consists of a set of players  $N = \{1, ..., n\}$ , strategy sets  $S_i$  and utilities  $u_i : S \to \mathbb{R}$ .

#### Definition

Strategy profile  $s_1 \in S_1, ..., s_n \in S_n$  where no one can unilaterally improve her own utility is called a Nash equilibrium.

(Nash) equilibria suffer from

- Strong belief assumptions
- Non simultaneous change (rules, democracy, marriage, traffic)
- Lack of coordination

## Solutions, Transitions and Efficiency - Plan

#### (Nash) equilibria suffer from

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- Non simultaneous change (democracy, marriage, traffic)
- Lack of coordination

No theoretical modelling of using various solutions simultaneously



#### $\Rightarrow \mathsf{We}$

- formally model a
  - a transition
  - a limited transition
  - a stable transition
- 2 bound efficiency

## Solutions, Transitions and Efficiency - Model - Transition

#### Definition

Given  $D \subseteq S$ , a transition is any profile  $s = (s_1, ..., s_n) \in S$  such that for each  $i \in N$ , there exists a solution  $d(s, i) = (d_1, ..., d_n) \in D$ , such that  $s_i = d_i$ .

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A stable transition is a transition, where no improvement is "straight-forward".

Disallow transitions that can be easily improved



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Coordination game on graphs, where any node chooses in  $\{r, b\}$ :



Disallow transitions where a player can increase her utility, regardless the others' unilateral improvements

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## Solutions, Transitions and Efficiency - Model - Efficiency

#### Definition

The social welfare is the sum of the utilities, i.e.

$$\mathsf{SW}(s) \stackrel{\Delta}{=} \sum_{i \in \mathsf{N}} u_i(s)$$

#### Definition

We look at  
PoA 
$$\stackrel{\Delta}{=} \frac{\min_{s \in D} SW(s)}{\max_{s \in S} SW(s)}$$
 and PoS  $\stackrel{\Delta}{=} \frac{\max_{s \in D} SW(s)}{\max_{s \in S} SW(s)}$ .  
PoTA  $\stackrel{\Delta}{=} \frac{\min_{s \in T(D)} SW(s)}{\max_{s \in S} SW(s)}$  and PoTS  $\stackrel{\Delta}{=} \frac{\max_{s \in T(D)} SW(s)}{\max_{s \in S} SW(s)}$ .  
 $m - PoTA \stackrel{\Delta}{=} \frac{\min_{s \in T(D,m)} SW(s)}{\max_{s \in S} SW(s)}$  and  $m - PoTS \stackrel{\Delta}{=} \frac{\max_{s \in T(D,m)} SW(s)}{\max_{s \in S} SW(s)}$ .  
PoSTA  $\stackrel{\Delta}{=} \frac{\min_{s \in ST(D)} SW(s)}{\max_{s \in S} SW(s)}$  and PoSTS  $\stackrel{\Delta}{=} \frac{\max_{s \in ST(D)} SW(s)}{\max_{s \in S} SW(s)}$ .

The price of anarchy can only become worse, when the set grows. We provide opposite bounds based on how the individual utilities depend on coordination and how the total utility depends on the individual ones.



# Solutions, Transitions and Efficiency - Constant-Sum and Potential Games

These are intuitively opposite kinds of games

 $\begin{array}{l} \mbox{Constant-sum} \\ \mbox{Constant-Sum games have } \mbox{PoTA} = 1. \end{array}$ 

#### Potential

 $m - \text{PoTA} \ge \text{PoA} / m$ , and this is tight.



## Solutions, Transitions and Efficiency - Constant-Sum and Potential Games - Decomposition

Using "Flows and decompositions of games: Harmonic and potential games" by Candogan et al., we treat general games.



## Solutions, Transitions and Efficiency - Routing Games

- routing games
- equilibrium flow
- oprice of anarchy

#### Definition

A transition as a feasible flow that is positive only where there is an equilibrium with a positive flow



Figure: Having *n* parallel edges with  $c_e(x) = x$  each.

We tightly bound the efficiency of transitions

## Solutions, Transitions and Efficiency - Coordination Games

Consider coordination games with colours  $\{r, b\}$  for everyone

#### Good Restriction

For the star topology with n-1 leaves, the only stable transitions that are not NE colour the cetre in r and  $\lfloor \frac{n-1}{2} \rfloor$  leaves in r and the rest  $\lfloor \frac{n}{2} \rfloor$  leaves in b, or fully switching between r and b.



#### Theorem

 $PoSTA \ge \frac{1}{2} - \frac{|N|}{2|E|}$ ,  $PoA \ge 1/2$ , and these bounds are tight.

## Help in Defending against Attacks - Conclusions

- Modelling lack of coordination
- **2** General efficiency bounds are appalling  $\Rightarrow$  coordinate
- On the bounds are optimistic for
  - certain routing games
  - constant-sum
  - limited transitions in potential games
  - stable transitions in coordination games (the denser, the better)

Removing non-equilibria stable transitions



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And SARNET?

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#### Superrational equilibria

- 9 Hofstader's superrational equilibria predict everyone will comply
- $@ \Rightarrow$  Confusing NE and superrational allows for all the possible profiles!



- When using trust and smart policies, what are the transitions?
- Further requirements/probabilities on transitions (SARNET)
- Is the given profile a (limited, stable) transition?



